Electronic Chaotic Oscillator Realization with Potential Uses in Communication Systems

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Overview

- Introduction
  - What is chaos?
- Background
  - Applications
  - Hybrid chaotic system
  - One-dimensional map
- Design approach
- Simulation results
- Hardware results
- Conclusion

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What is Chaos?

- Topological mixing
  - System evolves over time
  - Continuous spectral power density
- Long-term aperiodic behavior
  - No pattern
- Sensitivity to initial conditions (IC's)
  - Two very similar IC's result in very different trajectories
- Deterministic
  - Future behavior is fully determined by complete knowledge of initial condition

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Applications

- How is chaos electronics useful?
  - Enhanced functionality with minimal circuit complexity
- Communication systems
  - Continuous spectral power density
- Random Number Generator (RNG)
- Vehicle ranging and collision detection
- Noise signal generation
- Device characterization
  - BIST

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Hybrid Chaotic System

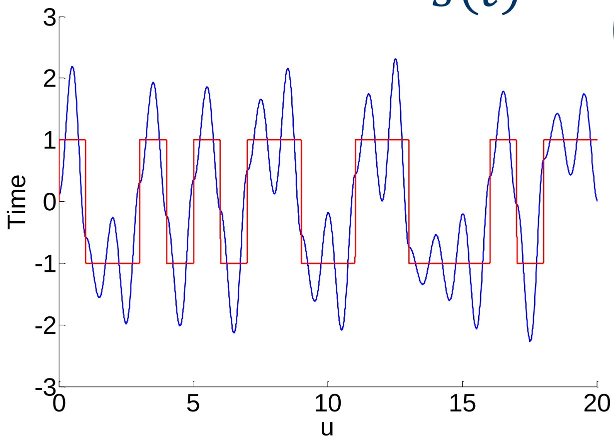
- Synthesized from a second order Linear continuous system and a piecewise switching event

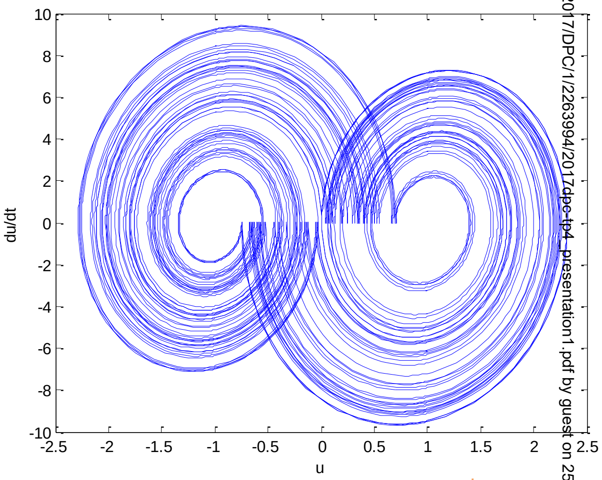
$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t)$$

- β – positive Lyapunov exponent
- Switching event:

$$s(t) = \begin{cases} +1, & \dot{u} = 0, u \geq 0 \\ -1, & \dot{u} = 0, u < 0 \end{cases}$$

Guard Condition





T. Saito, H. Fujita, "Chaos in a manifold piecewise linear system", Electronics and Communications Journal, vol. 64, no. 9, pp. 9-17, Oct. 1981.

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Exact Analytical Solution

Differential Equation:

$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t)$$

Solution:

$$u(t) = \sum_{m=-\infty}^{\infty} s_m P(t-m)$$

$$P(t) = \begin{cases} (1-e^{-\beta})e^{\beta t}\left(\cos\omega t - \frac{\beta}{\omega}\sin\omega t\right), & t < 0 \\ 1 - e^{\beta(t-1)}\left(\cos\omega t - \frac{\beta}{\omega}\sin\omega t\right), & t = 0 \\ 0, & t > 0 \end{cases}$$

- N. Corron, "An exactly solvable chaotic differential equation", Dynamics of Continuous, Discrete and Impulsive systems Series A: Mathematical Analysis, Jan. 2009.
- N. Corron, J. Blakely, M. Stahl, "A matched filter for chaos", Chaos, 20, 2010.

Exact Analytical Solution

- Why does this matter in a communication system?
  - Any practical transmission channel has noise
- Solution: Matched filter
  - Optimal linear filter for detecting a signal in the presence of additive white Gaussian noise (AWGN)

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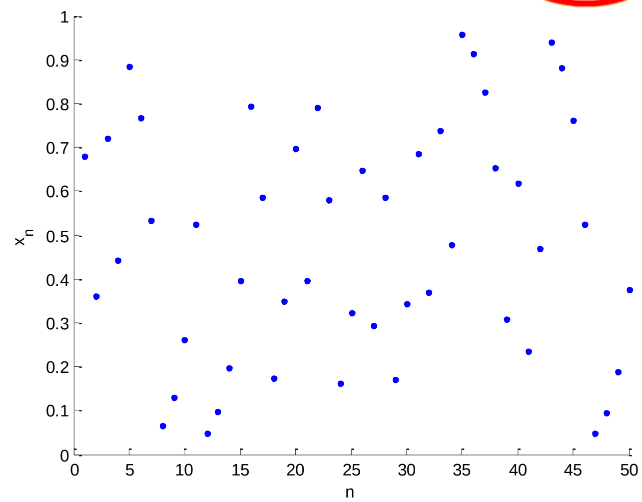
Iterated Shift Map

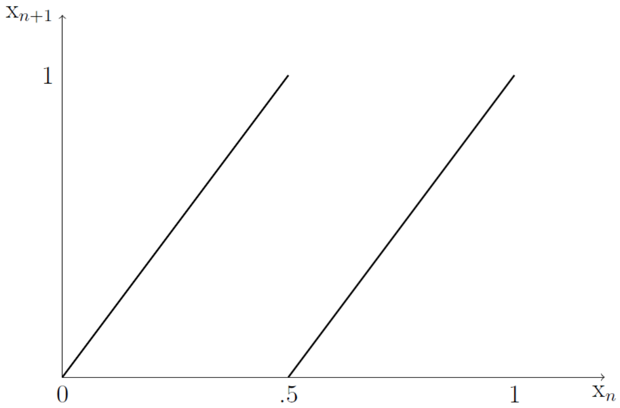
- One-dimensional map
- Bernoulli or Iterated shift map

$$x_{n+1} = 2\, x_n \, mod\, (1)$$
$$x_n \in (0,1]$$

- Conjugate to the Hybrid chaotic system

$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t)$$



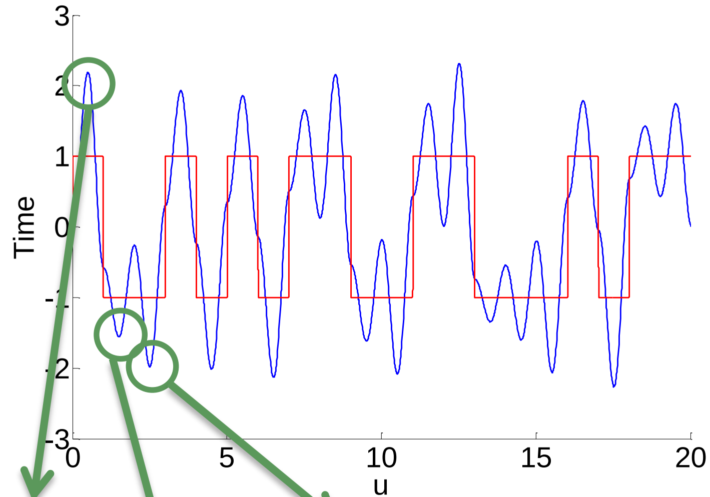


E. Ott."Chaos in dynamical systems", Cambridge University Press, Second Edition, 2002.

Relationship

- **Plot successive local max/min**

$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t)$$

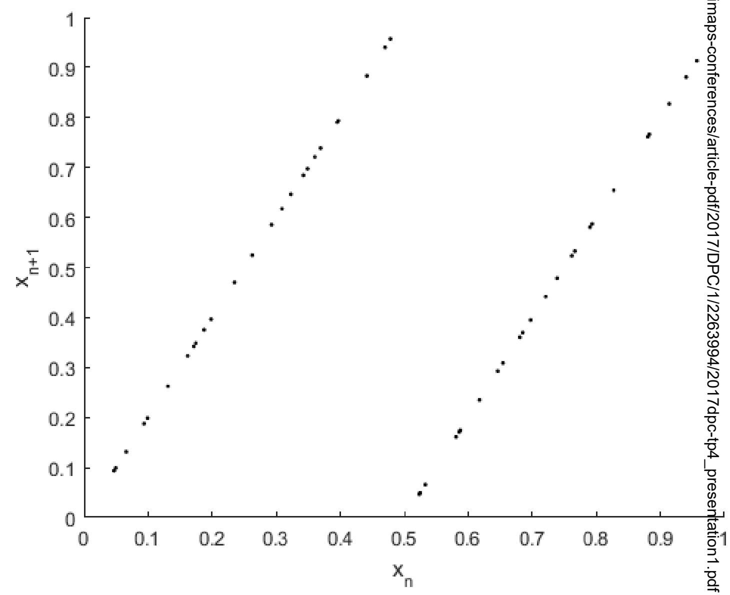


x_n x_{n+1} x_{n+2}

Normalize

$x_n \in (0,1]$

$$x_{n+1} = 2\, x_n \bmod (1)$$



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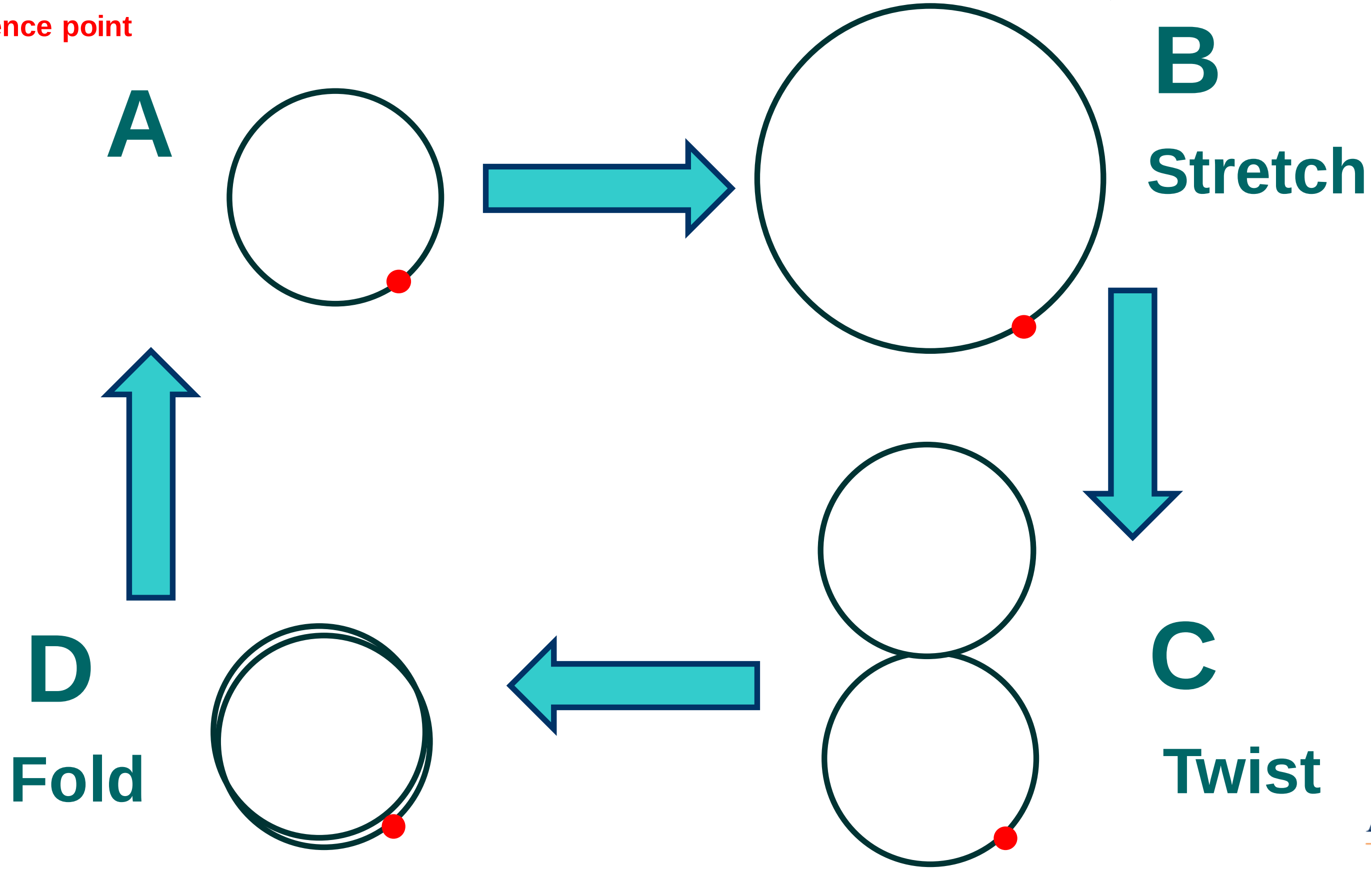
Simple Visual Representation of Shift Map

$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t) \qquad s(t) = \begin{cases} +1, & \dot{u} = 0, u \geq 0 \\ -1, & \dot{u} = 0, u < 0 \end{cases}$$

● - Reference point



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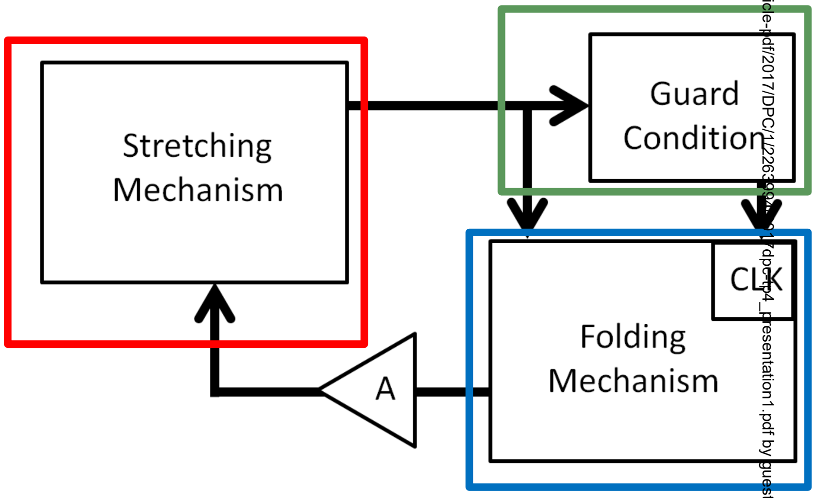
Design Approach

- Modular approach:
  - Three primary subsystems
    - Stretching mechanism (SM)
    - Guard condition (GC)
    - Folding mechanism (FM)



$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t)$$

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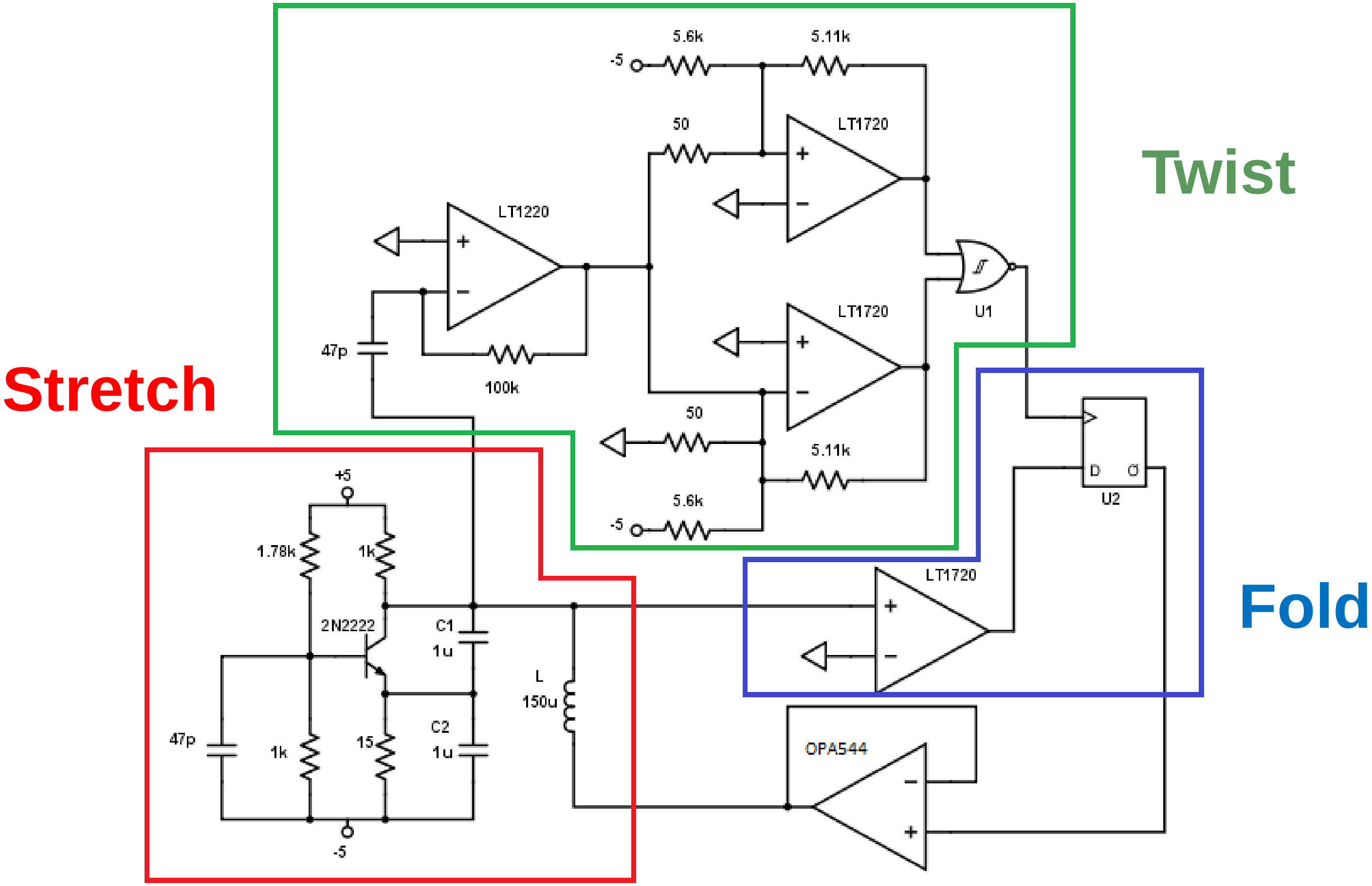
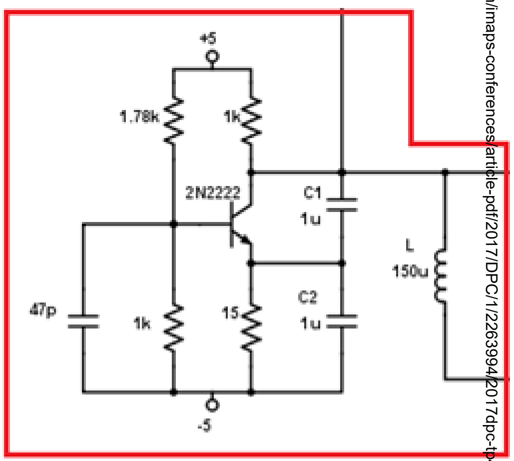
Generalized Schematic



Stretching Mechanism

Stretch

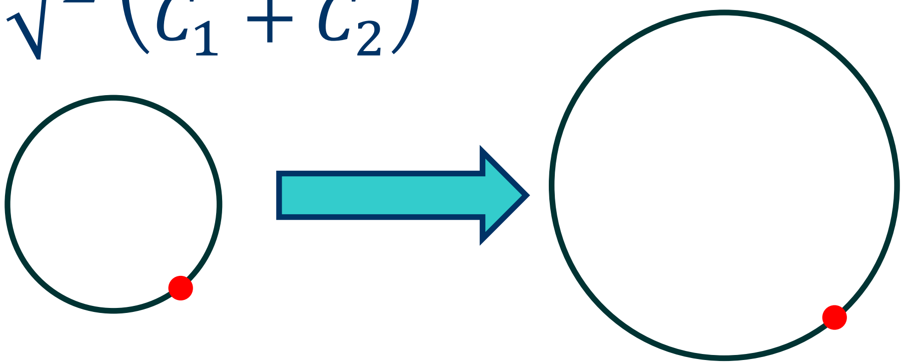


$$\ddot{u} - 2\beta\dot{u} + (\omega^2 + \beta^2)u = (\omega^2 + \beta^2)s(t)$$

- Common-base amplifier
  - β
- -RLC resonance

$$f = \frac{1}{2\pi\sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}} \approx 18.4\text{kHz}$$

A



B

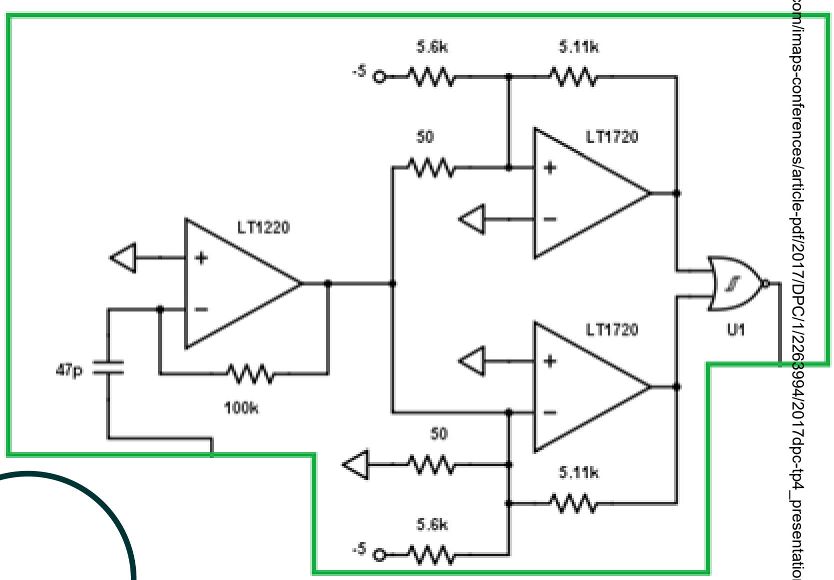
Stretch

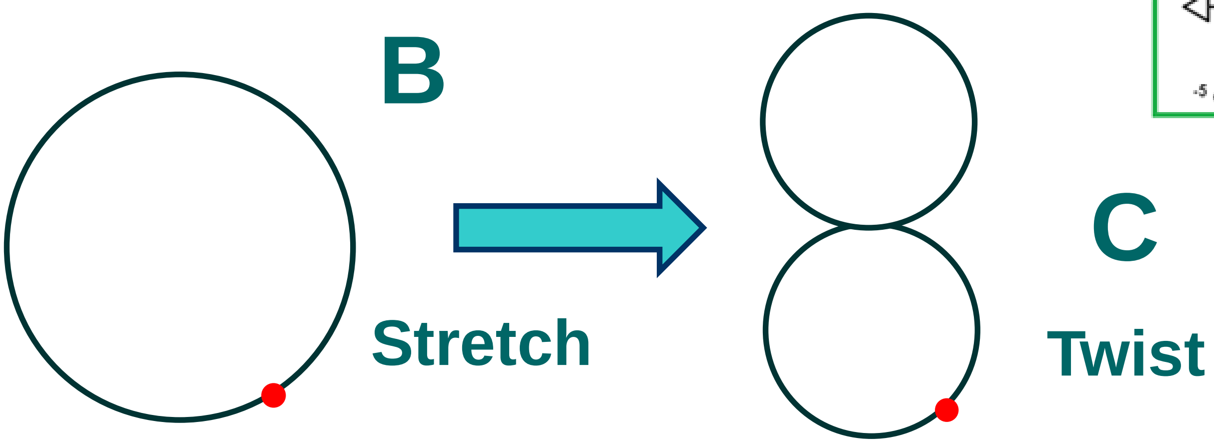
Twisting Mechanism

- Zero-crossing detection
- Guard condition

$$s(t) = \begin{cases} +1, & \dot{u} = 0, u \geq 0 \\ -1, & \dot{u} = 0, u < 0 \end{cases}$$

Twist

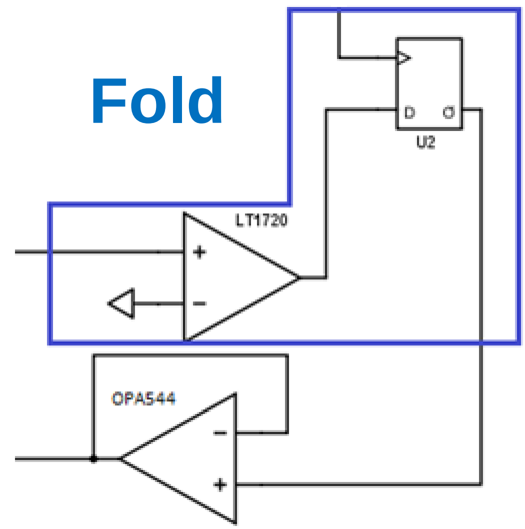


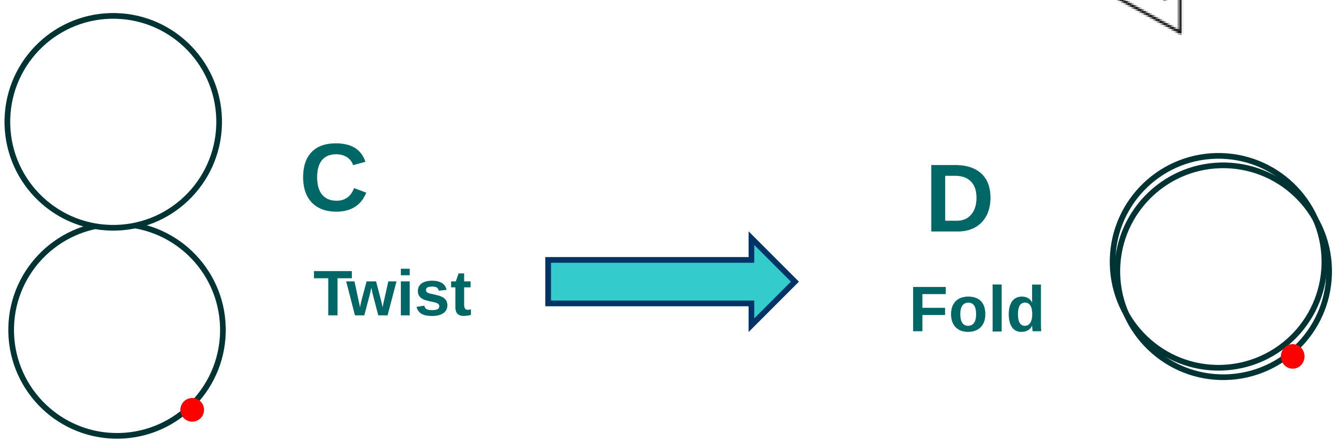


Folding Mechanism

- Sign detection
- Guard condition

$$s(t) = \begin{cases} +1, & \dot{u} = 0, \boldsymbol{u} \geq \boldsymbol{0} \\ -1, & \dot{u} = 0, \boldsymbol{u} < \boldsymbol{0} \end{cases}$$





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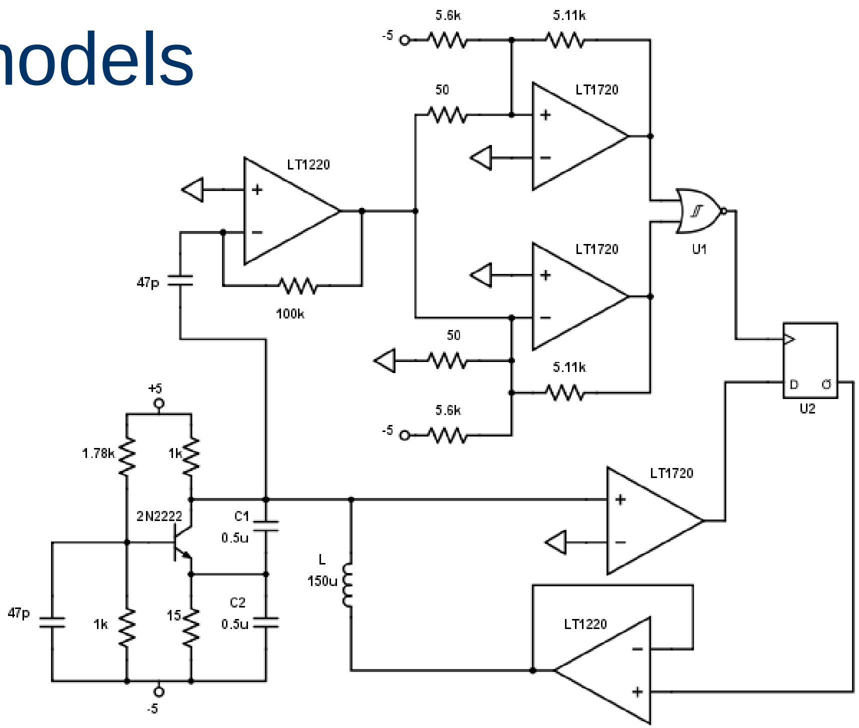
Simulation

- Schematic entered in SPICE
- COTS device models
  - LT1220
  - 2N2222
  - LT1720
  - 7400



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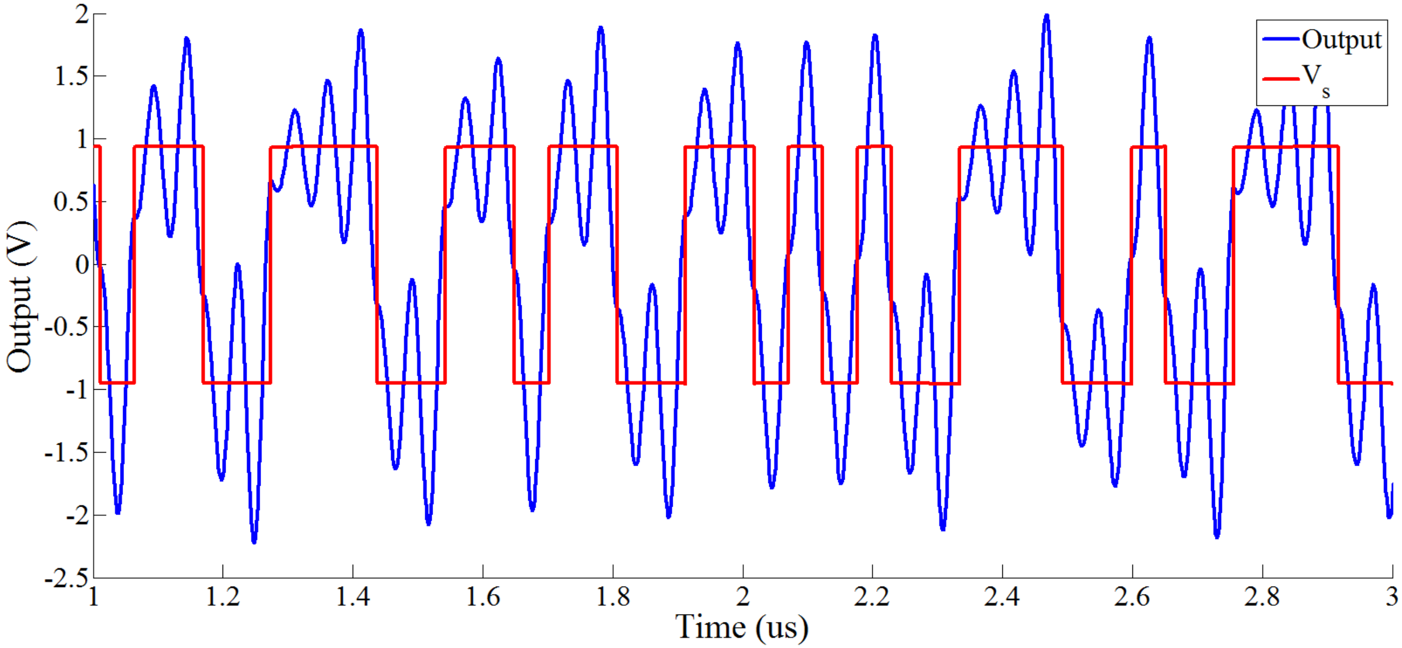
Simulation Results

- Long term aperiodic behavior
  - No pattern



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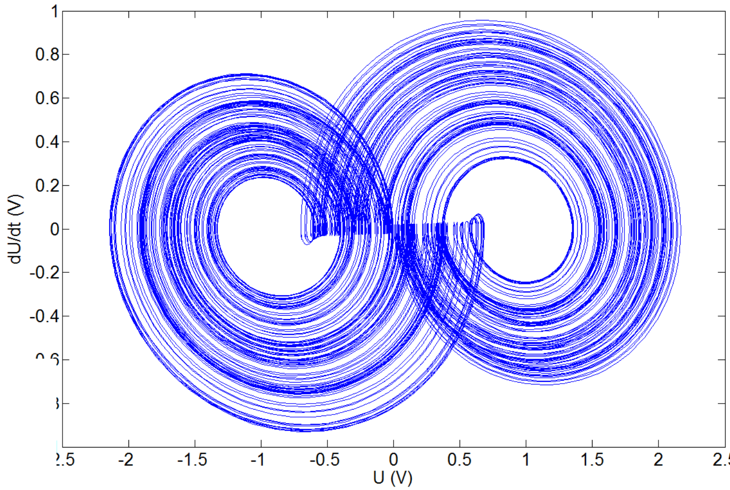
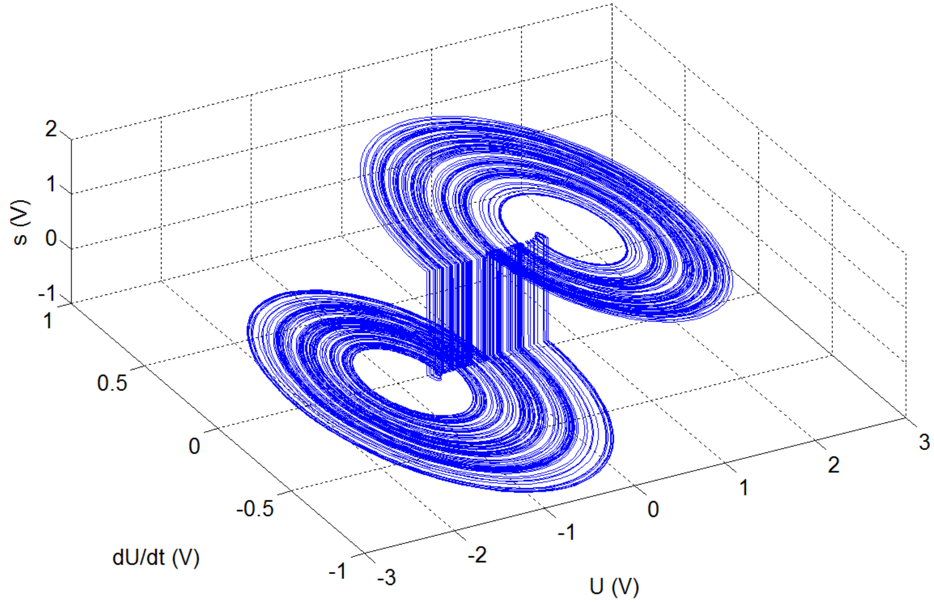
Simulation Results

- Topological mixing
  - System evolves over time
  - Dense attractor orbits





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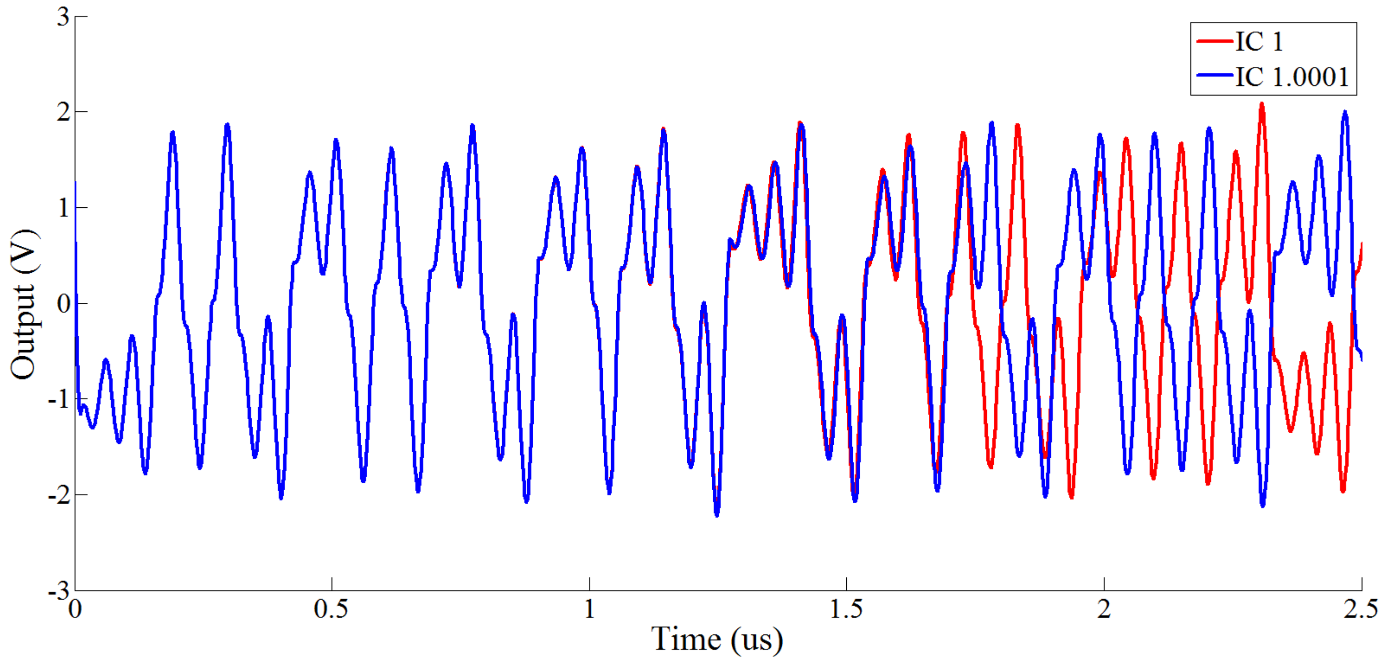
Simulation Results

- Sensitivity to IC's
  - Two very similar IC's result in very different trajectories



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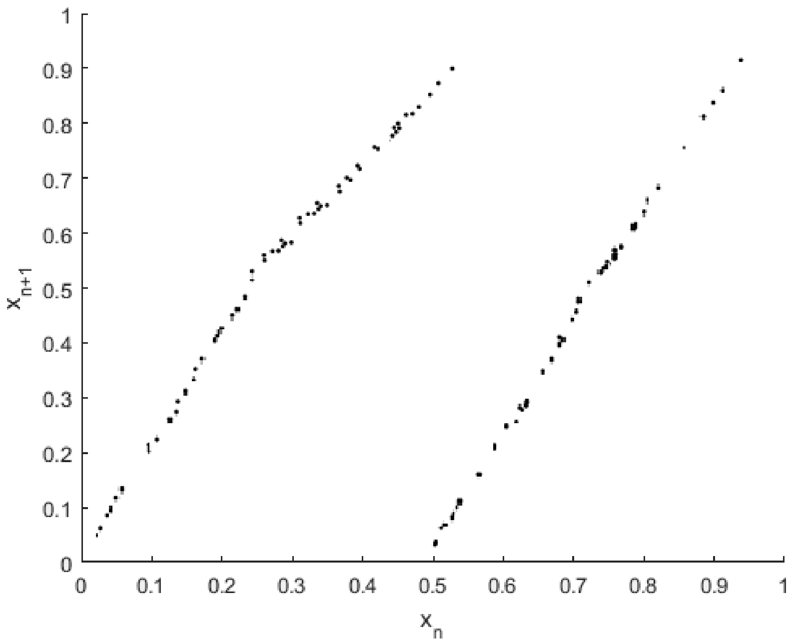
Simulation Results

- Successive local max/min
- Nonlinear recreation
  - Distortion in shift map
- Causes
  - Imperfect switching
  - Propagation delay



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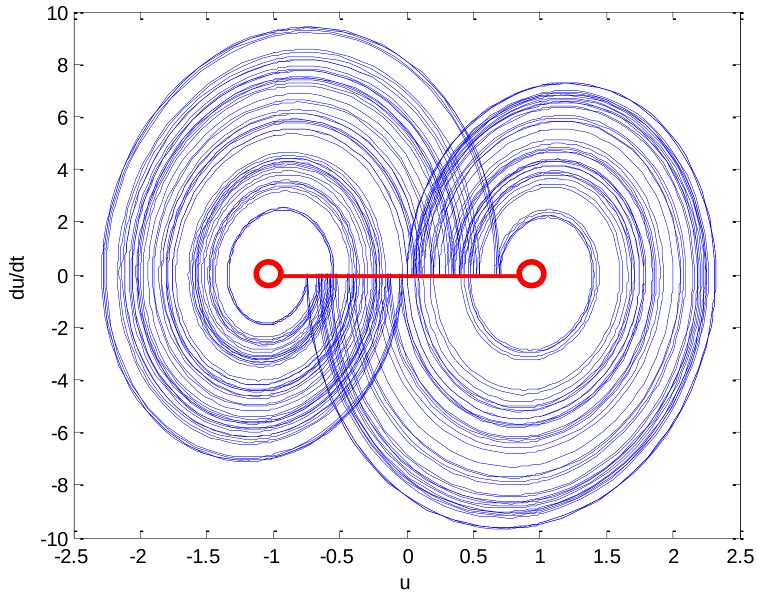
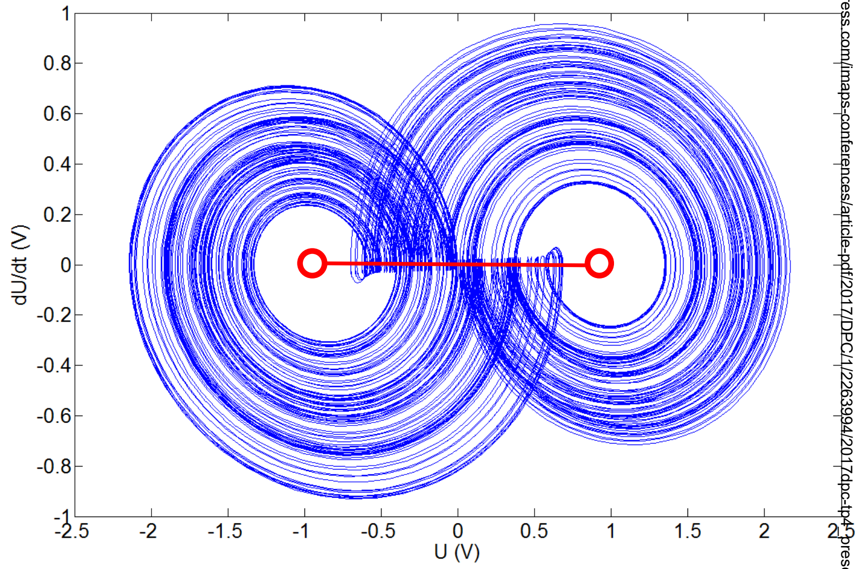
Distortion



Numerical Simulation



SPICE

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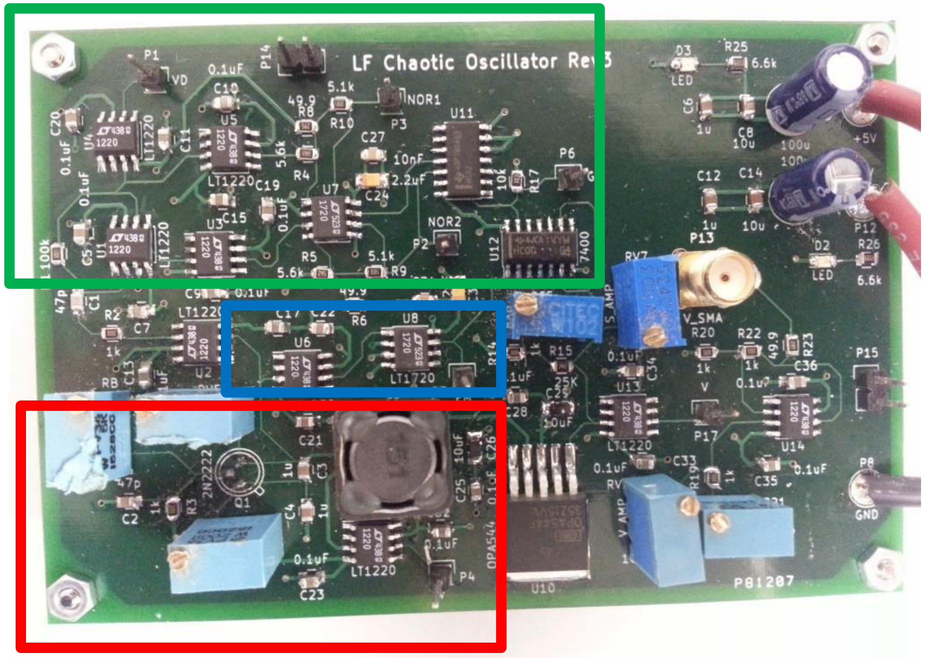
Electronic Implementation: Hardware

- Low Frequency Prototype
  - 18.4 kHz
  - COTS
  - Tunable
  - 50Ω SMA
  - 4” x 2.5”



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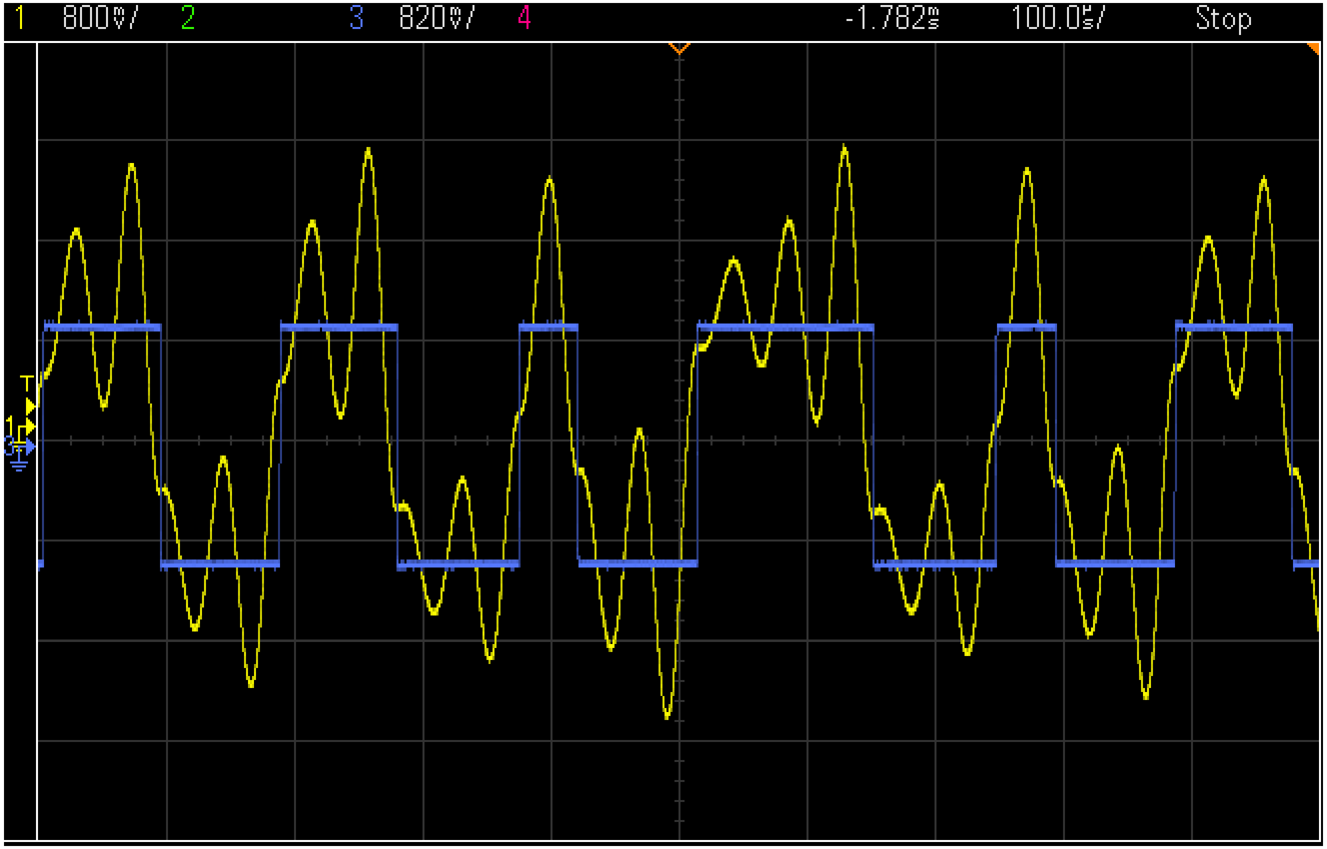
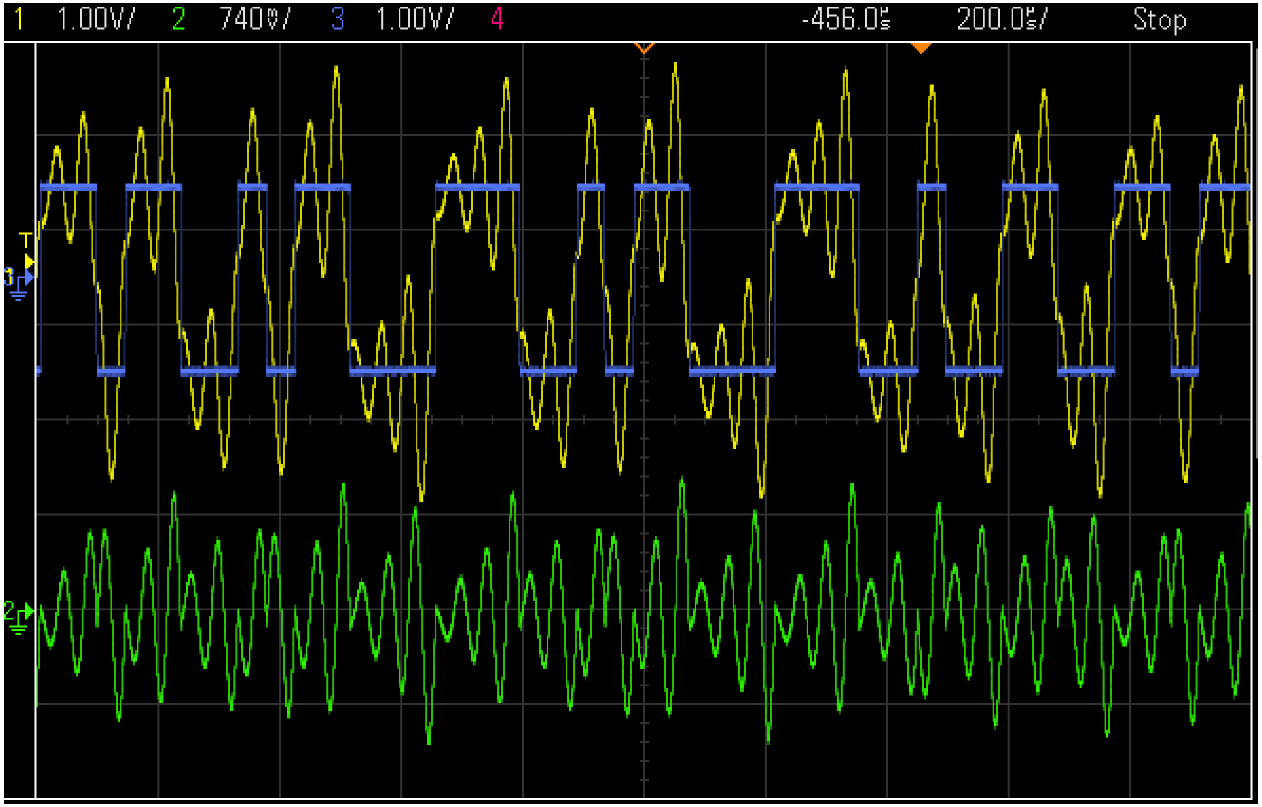
Time Domain Results



Output

s(t)

Time Domain Results



Output

s(t)

Derivative

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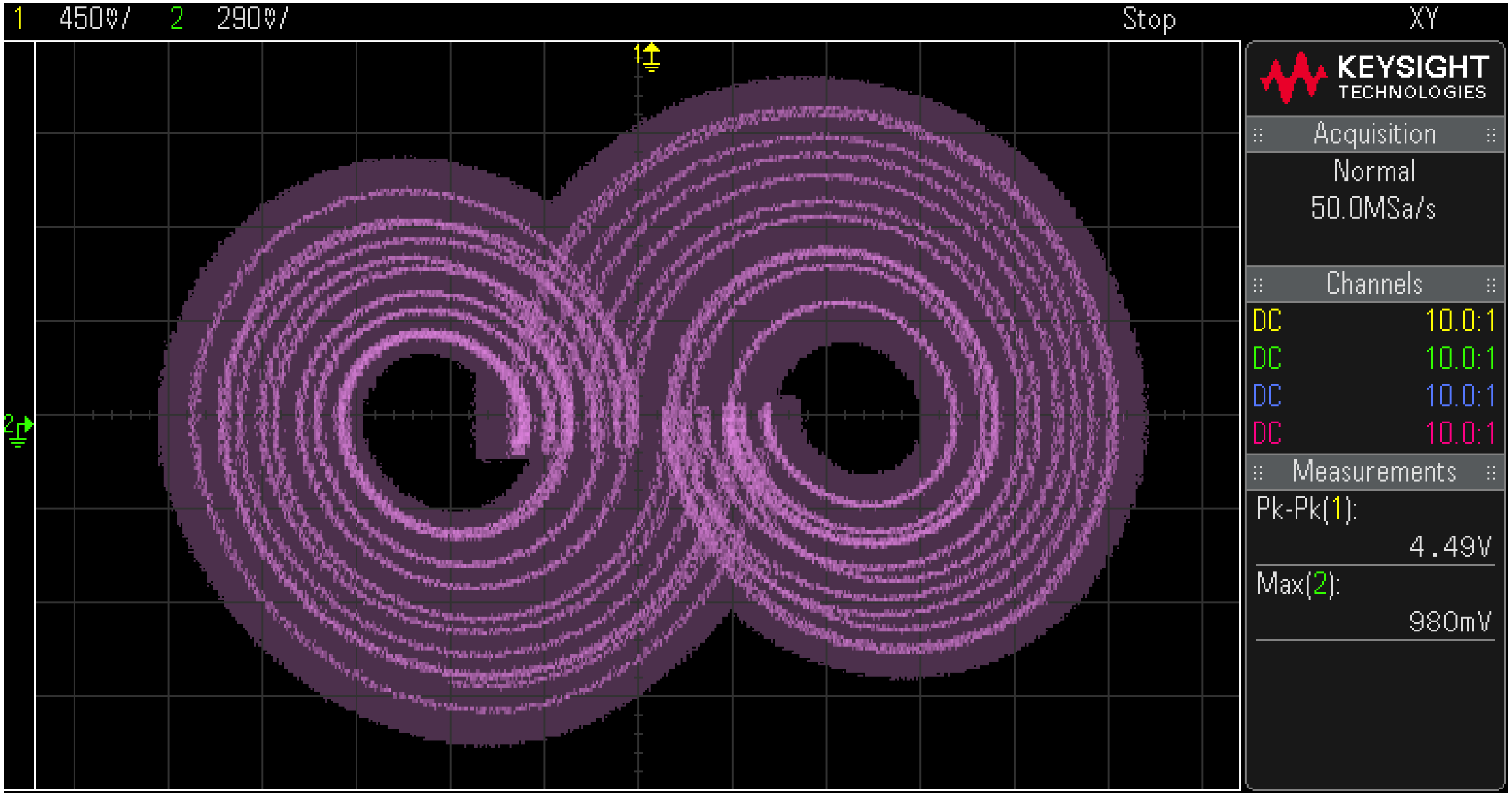
Phase Space



Derivative

Output

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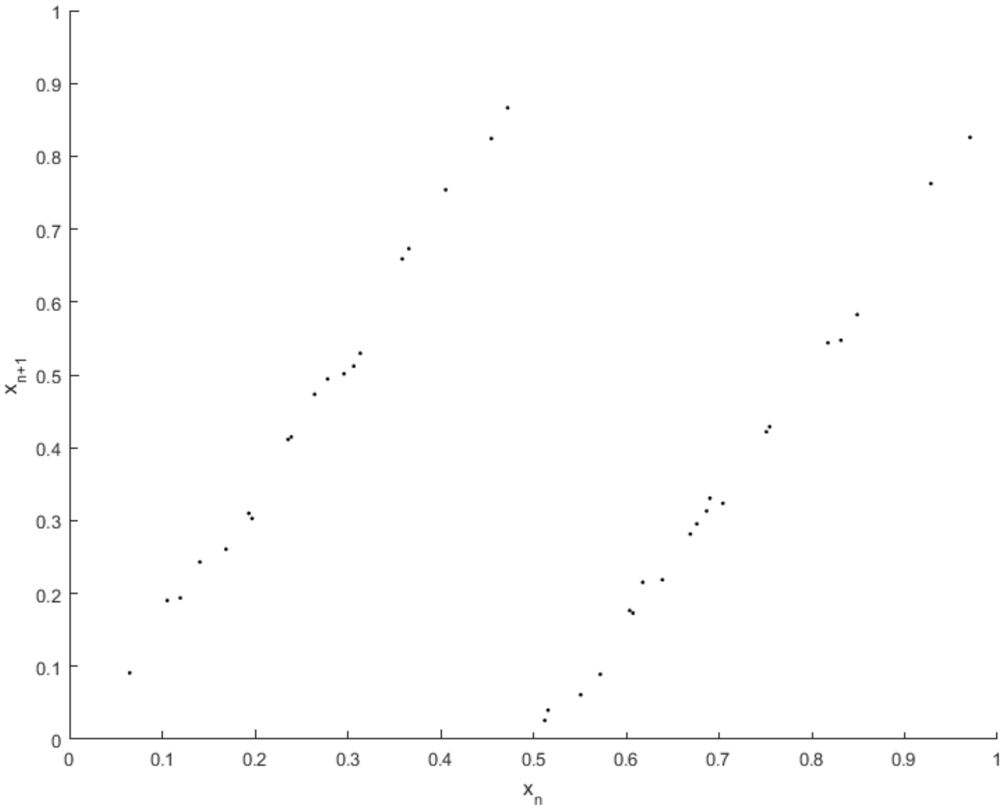
Shift map



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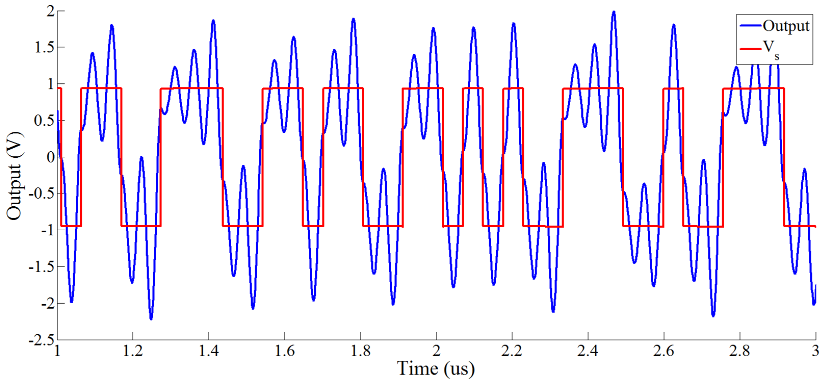
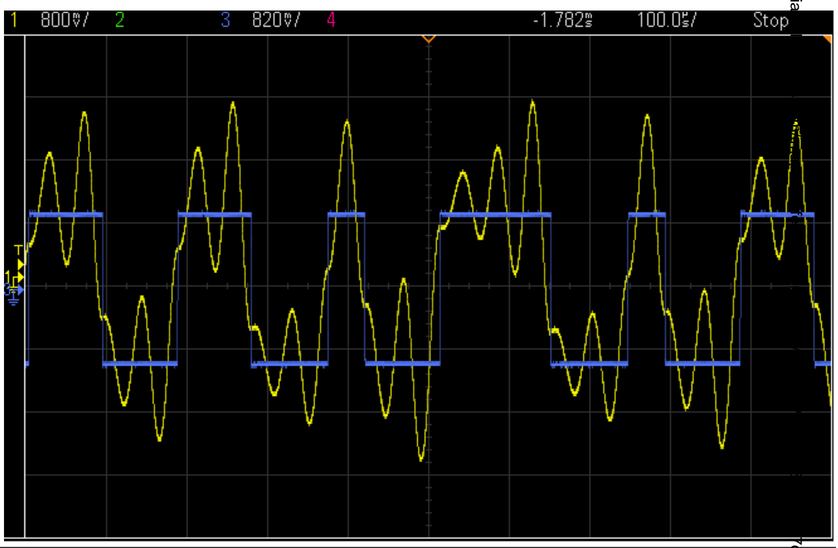
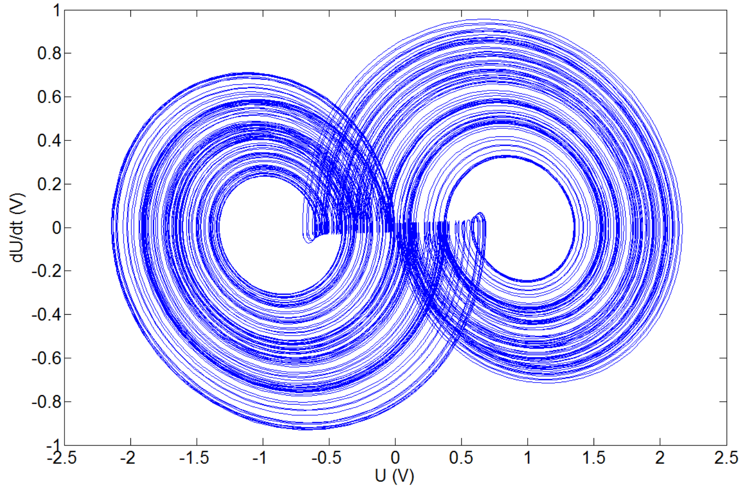
Simulation vs Hardware







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Conclusions

- Electronic implementation of a hybrid chaotic system
  - Potential use in communication systems
- Design approach
  - Subsystems
- Simulation and hardware results in high agreement with original system

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Questions?

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