Appendix A: Trig Functions of Special Angles

First we consider $\frac{\pi}{4}$.

$$sin\left(\frac{\pi}{4}\right) = cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = cos\left(\frac{\pi}{4}\right)$$

Since

$$1 = sin^2\left(\frac{\pi}{4}\right) + cos^2\left(\frac{\pi}{4}\right) = 2sin^2\left(\frac{\pi}{4}\right)$$

we obtain

$$sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = cos\left(\frac{\pi}{4}\right)$$

To find values for $\sin\frac{\pi}{6}$, we need the triple-angle identity

$$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$$

This follows from expanding $\sin(2\theta + \theta)$. We can now write

$$1 = \sin\frac{\pi}{2}$$

$$= \sin\left(3 \cdot \frac{\pi}{6}\right)$$

$$= 3\sin\frac{\pi}{6} - 4\,sin^3\frac{\pi}{6}$$

We solve this cubic equation in $\sin\frac{\pi}{6}$ to obtain a double solution $x = ½$ and single solution $x = -1$. Because $\sin\frac{\pi}{6} > 0$, we must have

$$sin\frac{\pi}{6} = \frac{1}{2}$$

And so

$$cos\,\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Here is a summary table:

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin x	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos x	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

Supplemental Material for:
Gresham, J., B. Wyatt & J. Crawford. 2019. Essential trigonometry without geometry. Texas J. Sci. 71: Article 10. https://doi.org/10.32011/txjsci_71_1_Article10.